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Vortex correlation functions in Maxwell–Chern–Simons models

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Abstract

Maxwell–Chern–Simons models in the presence of an instanton–anti-instanton background are studied. The saddle-point configuration corresponds to the creation and annihilation of a vortex localized around the Dirac string needed to support the nontrivial background.

This configuration is generalized to the case in which a nonlocal Maxwell term is allowed in order to fulfill the finite action requirement.

Following 't Hooft procedure, we compute the vortex correlation functions and we study the possibility of obtaining spin 1/2 excitations. A possible connection with the bosonization of interacting three-dimensional massive fermionic systems is also discussed.

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1. Introduction

Bosonization is an important tool to study interacting fermionic systems. Concerning the case of parity breaking models in $(2+1)D$, many efforts are being undertaken in order to improve this program. In particular, it is well established that the correlation functions of $U(1)$ fermionic currents correspond to correlation functions of topological currents in the dual bosonized theory [1,2]. This feature holds for both $(1+1)D$ and $(2+1)D$ models and has a universal character [3], as stated by the following formula

$$K_F[\psi] + I[j^F] \leftrightarrow K_B[\lambda] + I[\varepsilon\partial\lambda], \quad (1.1)$$

where K_F stands for the free fermionic action and K_B is the corresponding bosonized version. The term $I[j^F]$, with $j_\mu^F = \bar{\psi}\gamma_\mu\psi$, represents a generic current interaction. The bosonizing field λ is a scalar field ϕ in $(1+1)D$, and a vector field A_μ in $(2+1)D$. Accordingly, $\varepsilon\partial\lambda$ has to be read as $\varepsilon_{\mu\nu}\partial_\nu\phi$ or $\varepsilon_{\mu\nu\rho}\partial_\nu A_\rho$, respectively. It is worth mentioning here that the mapping (1.1) provides a unifying framework to derive universal transport properties of both one and two-dimensional interacting fermionic systems [4].

Similarly to the $(1+1)D$ case, where fermions can be associated to soliton configurations in the dual massive sine-Gordon theory [5], one would like to understand the elementary fermionic modes in $(2+1)D$ in terms of topological excitations in the bosonized dual theory. The latter is a gauge theory whose quadratic part is given by a nonlocal Maxwell–Chern–Simons (MCS) term [1–3]. In particular, when

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a large mass expansion is performed, the dominant term reduces to the usual local MCS action, namely

$$S(A) = \int d^3x \left(\frac{1}{2m} \mathcal{F}_\mu^2 + \frac{i}{2\eta} A_\mu \mathcal{F}^\mu \right), \quad (1.2)$$

where m is proportional to the fermion mass and η is the Chern–Simons coefficient in the fermionic effective action.

In $(2+1)D$, it is a common wisdom to believe that fermions should be related to vortices in the dual theory. The aim of this Letter is to pursue this investigation. Combining 't Hooft approach [6] to the quantization of extended objects in euclidean spacetime with the Hennaux–Teitelboim work [7] on instantons in MCS theory, we shall be able to show that vortices may appear as excitations with definite mass and spin in a generalized MCS model. The relationship among vortices in MCS and fermionic excitations will be analysed through Polyakov's spin action for Bose–Fermi transmutation in $(2+1)D$ [8]. 't Hooft framework is particularly adapted whenever the Mandelstam operators are not known. As an example, it has been successfully used to obtain a covariant quantization for the soliton excitations of the Skyrme model [9]. We also point out that the finite action requirement for vortex configurations is fulfilled by introducing a suitable nonlocal Maxwell term.

The present Letter is organized as follows. In Section 2 we study MCS vortex solutions in the presence of an instanton–anti-instanton background. Section 3 is devoted to the vortex quantization through the corresponding correlation functions and to the analysis of Polyakov's term. In Section 4, the nonlocal MCS case is discussed.

2. Vortices in Maxwell–Chern–Simons

In recent works [10,11] the existence of vortex solutions in Maxwell–Chern–Simons (MCS) in the presence of singularities has been discussed. These singularities turn out to be related to the continuum limit of a compact lattice version of the theory. The resulting classical solution to the equations of motion displays the behavior of a vortex. Although this configuration could be interpreted as a kind of energy lump due to its fast decay given by the MCS

topological mass, the corresponding total energy has a mild logarithmic divergence in the ultraviolet region [11]. In addition, the vortex is pinned around the position of the singularity, which is introduced as an external fixed source. In order to promote this field configuration to a particle-like excitation we have to give translational degrees of freedom to the vortex and render its energy finite. Also, the vortex propagator should be well behaved, without unphysical modes.

Following 't Hooft procedure, the vortex propagation in euclidean space is obtained by integrating over configurations where a vortex excitation is created out of the vacuum at a spacetime point x_1 and after an intermediate propagation is annihilated at x_2 . Before x_1 and after x_2 the topological charge vanishes, while it is nonvanishing in between due to the existence of the vortex. Therefore, suitable instanton–anti-instanton singularities have to be introduced at x_1 and x_2 in order to match these inequivalent topological configurations. In the present three-dimensional case these singularities can be seen as a monopole–anti-monopole pair [7,12] for the dual field strength configuration $\mathcal{F}_\mu = (1/2)\varepsilon_{\mu\nu\rho} F^{\nu\rho}$, located at x_1 and x_2 , respectively. One possible action describing the coupling of this pair with the MCS field is given by

$$S(A, J) = \int d^3x \left(\frac{1}{2m} (\mathcal{F}_\mu + J_\mu)^2 + \frac{i}{2\eta} A_\mu \mathcal{F}^\mu \right), \quad (2.3)$$

with

$$J^\mu(x) = \int_\gamma dy^\mu \delta^3(x - y), \quad (2.4)$$

where γ is an open smooth string running from x_1 to x_2

$$\partial^\mu J_\mu = \delta^3(x - x_1) - \delta^3(x - x_2). \quad (2.5)$$

The equations of motion are easily worked out and yield [10]

$$\begin{aligned} \mathcal{F}_\mu^{\text{cl}} &= -J_\mu + \mathcal{R}_\mu, \\ \mathcal{R}_\mu &= \frac{1}{4\pi} \left(\frac{m^2}{\eta^2} \delta_{\mu\alpha} - i \frac{m}{\eta} \varepsilon_{\mu\alpha\beta} \partial^\beta \right) \int_\gamma dy^\alpha \frac{e^{-\frac{m}{\eta}|x-y|}}{|x-y|}. \end{aligned} \quad (2.6)$$

The term \mathcal{R}_μ in the above expression represents a vortex configuration propagating from x_1 to x_2 , having both magnetic and electric field. We observe

that, due to the presence of the exponential factor in Eq. (2.6), \mathcal{R}_μ is localized around the curve γ , on a scale of the order of $1/m$. We also note that the Bianchi identity $\partial^\mu \mathcal{F}_\mu^{\text{cl}} = 0$ implies that $\partial^\mu \mathcal{R}_\mu = \delta^3(x - x_1) - \delta^3(x - x_2)$. Therefore, the flux Φ of the nonsingular part \mathcal{R}^z of the magnetic field, computed through any constant time plane Σ located between x_1 and x_2 , is

$$\Phi = \int_{\Sigma} d^2x \mathcal{R}^z = \oint dS_\mu \mathcal{R}^\mu = 1, \quad (2.7)$$

where the second equality follows by closing Σ with the addition of a surface at infinity giving no contribution due to the exponential decay of \mathcal{R}_μ .

The static limit corresponds to a configuration where the vortex is created in the far past and annihilated in the far future, and it always sits at the same position, that is, the associated string γ is an infinite straight line along the euclidean time-axis, identified with the z -axis. In this case, Eq. (2.6) reproduces the vortex profile discussed in Ref. [11]. In particular, for the magnetic field we get

$$\mathcal{F}_z^{\text{cl}} = -\delta^{(2)}(x) + \frac{1}{2\pi} K_0\left(\frac{m}{\eta}\rho\right), \quad (2.8)$$

with K_0 being the Bessel function and ρ the radial coordinate in the (x, y) -plane. Also, the point-like singularity introduced in [11], where the vortex is pinned, is nothing but the intersection of the string with the constant time plane Σ .

3. Quantization of the MCS vortices

Following 't Hooft prescription [6], in order to compute the vortex propagator we have to path integrate over all physical inequivalent configurations representing the creation, propagation and annihilation of the vortex. Therefore, we integrate over the gauge fields and all possible strings, and define the two-point vortex correlation function as

$$\begin{aligned} \mathcal{G}(x_1 - x_2) &= \int D\gamma \int DA e^{-S(A, J)} \\ &= \int D\gamma e^{-\Gamma_\gamma}, \end{aligned} \quad (3.9)$$

where Γ_γ represents the effective action obtained by integrating over all gauge configurations in a fixed

string background. The presence of the measure $D\gamma$ is natural in a path integral approach [8], being in fact needed in order to ensure the string independence of $\mathcal{G}(x_1 - x_2)$. This prescription should guaranty the locality of the quantum vortex field operators whose expectation value has to be identified with $\mathcal{G}(x_1 - x_2)$, although, in general, a closed form for these operators is not known.

In the pure Maxwell case, corresponding to the limit $m \rightarrow 0$, Γ_γ turns out to be independent from the particular Dirac string joining the singularities [6]

$$\Gamma_\gamma^{\text{Max}} \propto \frac{1}{|x_1 - x_2|}, \quad (3.10)$$

meaning that here the string is not observable. The integration over the paths is now trivial and results in a pure normalization factor. The path-independence of $\Gamma_\gamma^{\text{Max}}$ allows us to deform the original γ into two strings γ_1, γ_2 , where γ_1 goes from x_1 to ∞ and γ_2 from ∞ to x_2 . In this case, the vortex correlation function in Eq. (3.9) can be written in terms of Mandelstam variables $\mu(\gamma_1), \bar{\mu}(\gamma_2)$, according to

$$\begin{aligned} \mathcal{G}^{\text{Max}}(x_1 - x_2) &= \mathcal{N} \int DA \mu(\gamma_1) \bar{\mu}(\gamma_2) e^{-\frac{1}{2m} \int d^3x \mathcal{F}^2}, \\ \mu(\gamma_1) &= e^{-\frac{1}{m} \int_{\gamma_1} dx^\mu \mathcal{F}_\mu}, \\ \bar{\mu}(\gamma_2) &= e^{-\frac{1}{m} \int_{\gamma_2} dx^\mu \mathcal{F}_\mu}. \end{aligned} \quad (3.11)$$

The string independence of the effective action (3.10) corresponds to the well-established locality properties of the Mandelstam operators, in models containing pure Maxwell terms [13]. Coming back to the MCS case, it is easy to convince oneself that the effective action Γ_γ in Eq. (3.9) has a nontrivial dependence on γ . Therefore, as the string is now observable, we have to integrate over all paths, according to the general definition (3.9). On physical grounds, this amounts to take into account all possible intermediate processes representing the vortex propagation. We underline that in this case an explicit expression for the vortex operators is not available. However, the knowledge of the vortex propagator is sufficient to characterize the physical properties of the vortex at the quantum level.

As the integration over the gauge fields in Eq. (3.9) is quadratic, we obtain

$$\Gamma_\gamma = S(A^{\text{cl}}, J), \quad (3.12)$$

where A^{cl} is a vector potential for the saddle point configuration \mathcal{F}^{cl} in Eq. (2.6). After performing the spacetime integral, Γ_γ can be cast in the form of a double-line integral over the curve γ , with a kernel which is found to be localized on a scale of the order of $1/m$ (see Eq. (4.22) in Section 4). For well separated x_1 and x_2 , and smooth strings, the effective action Γ_γ , up to order $1/m$, is

$$\Gamma_\gamma \sim \lambda m L + \frac{\text{const}}{m} \int_0^L ds \frac{de^\alpha(s)}{ds} \frac{de^\alpha(s)}{ds}, \quad (3.13)$$

where L is the length of the curve γ , $e^\alpha(s)$ is the tangent vector dy^α/ds and the parameter s is defined through the relation $e^\alpha(s)e^\alpha(s) = 1$. The factor λ is logarithmic divergent [11], and will be discussed in the next section.

Notice that the presence of the second term in (3.13) is in fact already known [8] and takes into account velocity correlations at different points along γ . In order to obtain the vortex propagator $\mathcal{G}(x_1 - x_2)$ it remains to perform the integration over all possible paths γ with fixed end-points. This integration can be found in [8], yielding as final result the Klein–Gordon propagator.

The spinless character of this excitation is due to the complete cancellation of all imaginary terms of the kind

$$S_\gamma = \frac{1}{4\pi} \int_\gamma dx^\alpha \int_\gamma dy^\beta \varepsilon_{\mu\alpha\beta} \partial_\mu^x \frac{1}{|x - y|}, \quad (3.14)$$

arising from the presence of the Chern–Simons action. Observe that, for closed γ , this expression is known as the self-linking of the curve.

It is worth underlining that, depending on the coupling between the string and the MCS gauge potential, different kinds of correlation functions will be obtained, leading to different quantum numbers for the corresponding vortex excitations. For instance, if instead of (2.3) one considers the more general coupling

$$S(A, J) = \int d^3x \left(\frac{1}{2m} (\mathcal{F}_\mu + J_\mu)^2 + \frac{i}{2\eta} A_\mu \mathcal{F}^\mu + i \vartheta A_\mu J^\mu \right), \quad (3.15)$$

for the leading terms of the effective action Γ_γ one gets

$$\Gamma_\gamma \sim \lambda m L + \frac{i}{2} \eta \vartheta^2 S_\gamma. \quad (3.16)$$

In particular, for $\eta \vartheta^2 = 2\pi$, Polyakov’s Bose–Fermi transmutation occurs and the vortex propagator turns out to be that of a spin one-half fermionic excitation [8,14]

$$\int d^3p \frac{1}{\sigma^\mu p_\mu + \lambda m} e^{ip(x_1 - x_2)}, \quad (3.17)$$

where σ_μ are the Pauli matrices. With respect to the spinor index structure of this propagator we refer the reader to the original work [8]. In this regard, it is useful to point out that the functional integration in Eq. (3.9) should be equipped with appropriate fixed boundary conditions around the monopole–anti-monopole singularities, carrying a representation of the rotation group. At the locations of these singularities vortices with given quantum numbers will be created and destroyed. This will lead to the correct index structure for the final expression of the propagator. This framework has been worked out in Ref. [9] in the case of skyrmions.

4. Vortices in nonlocal MCS models

So far, we have seen that vortex configurations are present in MCS theory when a nontrivial instanton–anti-instanton background is introduced. Depending on the coupling with the string, the vortex quantum numbers may correspond to a bosonic or a fermionic excitation. However, as it has been already pointed out in [11], the energy of this configuration displays an ultraviolet logarithmic divergence. The aim of this section is to face this problem. One possibility in order to have a finite action configuration is that of introducing nonlocal terms in the action, whose effect is that of properly regularizing the ultraviolet region. For instance, this can be done by modifying the Maxwell term in (3.15) according to

$$S(A, J) = \int d^3x \left(\frac{1}{2} (\mathcal{F}_\mu + J_\mu) \widehat{O} (\mathcal{F}^\mu + J^\mu) + \frac{i}{2\eta} A_\mu \mathcal{F}^\mu + i \vartheta A_\mu J^\mu \right), \quad (4.18)$$

where \widehat{O} is a nonlocal operator associated with a kernel $O(x-y)$

$$[\widehat{O}\mathcal{F}](x) = \int d^3y O(x-y)\mathcal{F}(y).$$

We also require that the Fourier transform

$$\widetilde{O}(k) = \int d^3x e^{-ikx} O(x) \quad (4.19)$$

is positive definite.

The local Maxwell term is recovered by taking $O(x-y) = (1/m)\delta^{(3)}(x-y)$. We remark here that nonlocal MCS models appear in a natural way in the context of bosonization [2]. Indeed, these terms arise from the evaluation of the massive fermionic determinant in a generic background. We also observe that the presence of a current–current interaction in the starting fermionic action will produce in the bosonized action an additional nonlocal Maxwell term, which follows from the universal bosonization rule (1.1), namely

$$\begin{aligned} & \frac{1}{2} \int d^3x d^3y j_\mu^F(x) G(x-y) j_\mu^F(y) \\ & \Leftrightarrow \frac{1}{2} \int d^3x d^3y \mathcal{F}_\mu(x) G(x-y) \mathcal{F}^\mu(y). \end{aligned} \quad (4.20)$$

Coming back to the nonlocal MCS action (4.18), the corresponding classical vortex profile gets modified according to

$$\mathcal{F}_\mu^{\text{cl}} = -J_\mu + \frac{(1-\vartheta)}{1-\eta^2\widehat{O}^2\partial^2} (J_\mu + i\eta\widehat{O}\varepsilon_{\mu\nu\rho}\partial_\nu J_\rho). \quad (4.21)$$

Upon substitution of this expression in Eq. (4.18) one obtains

$$\begin{aligned} S(A^{\text{cl}}, J) &= \frac{i}{2}\eta\vartheta^2 S_\gamma + \frac{1}{2}(1-\vartheta\eta)^2 \int d^3x J_\mu \frac{\widehat{O}}{1-\eta^2\widehat{O}^2\partial^2} J_\mu \\ &+ \frac{i}{2\eta}(1-\vartheta\eta)^2 \int d^3x J_\mu \frac{\widehat{O}^2}{1-\eta^2\widehat{O}^2\partial^2} \varepsilon_{\mu\nu\rho}\partial_\nu J_\rho. \end{aligned} \quad (4.22)$$

We note that the real part of the action is positive. Also, in the static limit in which γ is an infinite straight line coinciding with the z -axis, the action per unit length turns out to be

$$\frac{1}{2}(1-\vartheta\eta)^2 \int \frac{d^2k}{(2\pi)^2} \frac{\widetilde{O}}{1+\eta^2\widetilde{O}^2\mathbf{k}^2}, \quad (4.23)$$

where the quantities in boldface correspond to the two-dimensional projection $k \rightarrow (\mathbf{k}, 0)$. In the local case ($\widetilde{O} = 1/m$) this expression contains a mild logarithmic ultraviolet divergence [11]. However, in the case where \widetilde{O} behaves in the uv region as k^α ($\alpha > 0$), the action per unit length is rendered finite, no matter how small α is.

5. Conclusions

Following 't Hooft procedure, we have studied vortex correlation functions in MCS models considering different couplings between the gauge fields and the string associated with the instanton–anti-instanton pair. This string arises in the continuum limit of a compact lattice version of the theory [10,11].

With the exception of the pure Maxwell type case, the string is observable. Therefore, we have defined vortex correlation functions by path integrating over both the gauge fields and the string. This corresponds to take into account the vortex translational degrees of freedom. It is the integration over the string which finally leads to a well behaved propagator, without unphysical poles.

Concerning the bosonization of $(2+1)D$ fermionic systems we remind that, for large m , the dominant term in the bosonized action corresponds to the local MCS [1]. Furthermore, we have been able to see that the coupling in Eq. (3.15) leads to a vortex excitation with spin 1/2, whenever the condition $\eta\vartheta^2 = 2\pi$ is satisfied. Although a direct derivation of the bosonization formula for fermion propagators has not yet been obtained, this result gives a strong indication that the elementary fermionic excitations correspond indeed to vortices in the dual theory.

These vortex configurations have been generalized to the case in which a nonlocal Maxwell term is present. We have shown that this kind of term could improve the ultraviolet behavior so as to render the vortex energy finite.

On the other hand, for $\eta\vartheta^2 = 2\pi$, the possibility of identifying vortex and fermionic correlation functions together with the universal bosonization rule (4.20) could give a useful framework to analyse the spectrum of the excitations for interacting fermionic systems. While in the local MCS case the localization of the vortex on a scale of the order $1/m$ leads to

the existence of a pole in the vortex propagator due to Eq. (3.16), in the nonlocal case, depending on the fermionic interaction kernel $G(x - y)$ in Eq. (4.20), the vortex profile (4.21) could spread out. This would imply the breaking of the validity of the long distance approximation (3.16). This may result in the absence of the pole in the propagator, meaning that the quasiparticle picture could be destabilized by the interaction among fermions.

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References

- [1] E.C. Marino, *Phys. Lett. B* 263 (1991) 63;
E. Fradkin, F.A. Schaposnik, *Phys. Lett. B* 338 (1994) 253;
F.A. Schaposnik, *Phys. Lett. B* 356 (1995) 39;
- C.P. Burgess, C.A. Lutken, F. Quevedo, *Phys. Lett. B* 336 (1994) 18.
- [2] D.G. Barci, C.D. Fosco, L.E. Oxman, *Phys. Lett. B* 375 (1996) 267–272.
- [3] D.G. Barci, L.E. Oxman, S.P. Sorella, *Phys. Rev. D* 59 (1999) 105012.
- [4] L.E. Oxman, E.R. Mucciolo, I.V. Krive, *Phys. Rev. B* 61 (2000) 4603;
D.G. Barci, L.E. Oxman, *Nucl. Phys. B* 580 (2000) 721.
- [5] S. Coleman, *Phys. Rev. D* 11 (1975) 2088;
S. Mandelstam, *Phys. Rev. D* 11 (1975) 3026.
- [6] G. 't Hooft, *Nucl. Phys. B* 138 (1978) 1.
- [7] M. Henneaux, C. Teitelboim, *Phys. Rev. Lett.* 56 (1986) 689.
- [8] A. Polyakov, *Mod. Phys. Lett. A* 3 (1988) 325;
A. Polyakov, in: E. Brézin, J. Zinn-Justin (Eds.), *Les Houches*, 1988.
- [9] M. Kruczenski, L.E. Oxman, *Nucl. Phys. B* 488 (1997) 513–538.
- [10] M.C. Diamantini, P. Sodano, C.A. Trugenberger, *Phys. Rev. Lett.* 71 (1993) 1969–1972.
- [11] G. Dunne, A. Kovner, B. Tekin, *Phys. Rev. D* 63 (2001) 025009.
- [12] R.D. Pisarski, *Phys. Rev. D* 34 (1986) 3851.
- [13] E.C. Marino, *Int. J. Mod. Phys. A* 10 (1995) 4311–4324;
E.C. Marino, *Mod. Phys. Lett. A* 11 (1996) 1985.
- [14] J. Grundberg, T.H. Hansson, A. Karlhede, U. Lindström, *Phys. Lett. B* 218 (1989) 321.